0. About SBS
SBS Transit is a major public transport operator in Singapore. It was formed in 1973 under the name Singapore Bus Services with the merger of three bus companies. By constantly investing in technology and training and upgrading its operations and people, it seeks to provide a world-class public transport system that is safe, comfortable, affordable, reliable and friendly.

1. Introduction
Background
A schematic representation of the current Provisioning System is shown in Figure 1.

Motivations
- Revised government regulations on bus reliability and availability, enforced by a monetary reward and penalty scheme
- Improvement of public welfare through better bus services

Challenges
- Limited capacity of warehouses
- Uncertainty over unplanned demand
- Spare parts obsolescence

2. Objectives
Estimate Component Life-span and Mean time to failure
Forecast future demand to enable early warning and pre-empt of low stock situations
Determine desired inventory level and Order Quantities while taking into account service level
Reduce Probability of Stock-out with emphasis on slow-moving items, Non-consignment stock and parts without preventive maintenance

3. Methodology
Contingent upon the Company's requirements and information available on the usage records of the items from SBS Transit, different methods are employed. With detailed usage records, we can perform Life-span Analysis, and forecast demand with Life-span and Age Information. With only monthly consumption data records, we can perform Time-series Analysis to forecast demand. Inventory Policy recommendations are based on revised demand predictions.

Life-span Analysis
We aim to establish a parametric model characterizing the item's life-span. We assume that the life-spans of materials of the same material-number independently follow a common distribution \( F(x) \).

The Maximum-Likelihood Estimate of \( \alpha \) to maximize the likelihood of observing the actual data given the parameters

\[
\hat{\alpha} = \sum_{i=1}^{n} \frac{x_i - \bar{x}}{n \cdot \bar{x}^2} \quad \text{Mean-square-error} \quad \text{Exponential Smoothing} \]

\[
\frac{1}{n} \sum_{i=1}^{n} \left( x_i - \hat{x}_i \right)^2 \quad \frac{1}{n} \sum_{i=1}^{n} \left( x_i - \bar{x} \right)^2
\]

Figure 2 illustrates the outcomes of the analysis with one of a consumption record data fitted to the Weibull family. The (1) dotted line represents the distribution fitted with only exact failure times and (2) the solid line represents the distribution fitted with exact and censored data, which results in an unbiased estimate of life-span. Both are overlaid on the histogram of exact failure times, 10% failure probability region is shaded.

Failure Prediction with Life-span and Age Information
For a particular material with age \( \alpha \), the probability of failure in the next \( T \) days is

\[
P(\alpha) = \int_{0}^{\alpha} f(x) \, dx
\]

With \( k \) buses using this particular material, the number of failures in the next \( T \) days follows approximately a Poisson distribution

\[
X \sim \text{Poisson} \left( \sum_{i=1}^{k} \int_{0}^{\alpha} f(x) \, dx \cdot (1 - F(\alpha)) \right)
\]

Figure 3 illustrates the outcome of the prediction (solid blue line) with life-span and Age Information derived from a set of material consumption data, with 95% prediction interval (shaded) and actual failure counts (solid line with points) overlaid.

Failure Prediction with Time-series Analysis
We employ Moving Mean (MM) and Exponential Smoothing (ES) Models to predict future demand. Model with least MSE is selected.

Model prediction

\[
\hat{F}(x) = \sum_{i=1}^{k} \frac{1}{n} \sum_{i=1}^{n} \left( x_i - \bar{x} \right)^2
\]

\[
\frac{1}{n} \sum_{i=1}^{n} \left( x_i - \hat{x}_i \right)^2
\]

Figure 4 serves to illustrate the output of demand forecasting with time-series analysis on a selected set of material, with prediction (solid blue line), 95% prediction interval (shaded) and actual failure counts (solid line with points) overlaid.

Inventory Policy Analysis
For Fast-moving materials, Ordering Quantities (OQ) is decided based on the Predictions of demand forecast over the next review period and Lead Time (LT). Balance-on-hand (BOH), Outstanding Back-Orders (OBO), and Target service level (1-\( \alpha \))

\[
OQ = (1 - \alpha) \cdot \text{LT} \cdot \text{BOH} + \text{OBO}
\]

For Slow-moving materials, Base-stock Model is implemented to perform one-for-one replenishment. Stock-up-to-level (SL) is set at the level such that

\[
\alpha = \Pr (D_{LT} > S)
\]

Figure 5 illustrates simulated inventory cycle with historical demand under recommended policy, in which review period is set at 30 days and lead time is set at 45 days, service level at 95%.

4. Comparisons and Results
Simulation study for a Fast-moving material was performed to compare the recommended Inventory Policy and the Heuristic Approach. Three notable improvements can be identified:

1. Service level is improved. With target service level set at 95%, Recommended Policy reduced the number of stock outs to 2 over the 60-month period, while Heuristic Approach yields multiple and prolonged stock-outs.

2. Recommended approach enables early warning and pre-empt build-up of inventory as illustrated in the initial 12 months period.

3. Stock-level variability under recommended policy is reduced compared to the Heuristic Approach. Average stock level is reduced. Level variability under recommended policy is reduced compared to the Heuristic Approach. Average stock level is reduced. Level variability under recommended policy is reduced compared to the Heuristic Approach. Average stock level is reduced.

5. Deliverables
Improved Inventory Policy
- Periodic Review for general items
- Continuous Review and 1-for-1 replenishment for slow-moving, high-value items
- Strong applicability across all parts, based on item classification
- Promote ease of understanding and use for quick implementation

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- Inferences on days-to-replacement, given failure probability before maintenance (c.f. Figure 7)
- Forecasting tools: demand predictions with time-series analysis (c.f. Figure 9)
- Automatic determination of Ordering Quantities (c.f. Figure 9)