CONSISTENT ROUTE FLOWS AND THE CONDITION OF PROPORTIONALITY

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ABSTRACT

User equilibrium (UE) static deterministic traffic assignments are widely used in travel forecasting practice. Forecasting analyses that are based on route flows are also quite common, even though it is well known that route flows are not unique under the UE conditions, unless additional condition is imposed, such as the condition of proportionality.

The purpose of the research described here is to examine the nature and magnitude of the differences in route flows in a practical setting, by applying several assignment tools that are commonly used in practice, as well as a research assignment tool (TAPAS), to a case study based on the network of the Chicago region. Select link analyses from the various solutions are compared with each other, showing that the differences are practically meaningful.

The condition of proportionality is evaluated for each solution by itself. The findings show that TAPAS can produce a solution that satisfies the condition of proportionality nearly perfectly. Implementations of the Frank-Wolfe algorithm produce solutions that reasonably satisfy proportionality, but needed levels of precision in terms of UE convergence require substantial computation times. Alternative commercially available assignment tools provide quick precision, but their solutions seem to deviate considerably from the condition of proportionality.

Keywords: user-equilibrium, route flows, proportionality.
1 INTRODUCTION

1 Travel forecasting involves many challenges including data collection, network representation, modeling assumption decisions and more. Once the assumptions are determined and the needed data are ready the remaining task of computing a forecast may present additional non-trivial challenges. In the case of the widely used model of static, deterministic, user-equilibrium (UE) traffic assignment, the main computational challenges are precision and route flows. The focus of this research is route flows. We believe that precision must be taken into account in order to put in context the results presented here. Therefore, a brief discussion of the issue of precision in UE traffic assignment is given in section 3, and referred to subsequently as appropriate.

The main issue with route flows is that they are not uniquely determined by the UE conditions. This well known issue can be demonstrated by the simple example in Figure 1, showing the total link flows in a solution that presumably perfectly satisfies the UE conditions, meaning that the cost of travel from node 2 to node 5 is exactly the same whether traveling through node 3 or through node 4. If we switch one vehicle from A that uses the segment through 3 with a vehicle from B that uses the segment through 4, the total link flows remain the same, so link costs remain the same and the perfect equilibrium situation also remains.

Considering the four routes in Figure 1, [A,1,2,3,5,C]; [A,1,2,4,5,C]; [B,1,2,3,5,C]; and [B,1,2,4,5,C] we can examine three route flow solutions: [25, 75, 15, 45]; [40, 60, 0, 60]; and [0, 100, 40, 20]. These three route flow solutions are quite different from each other, even though all three of them correspond exactly to the same total link flows shown in Figure 1.

In many practical applications, for example in most cost-benefit analyses, we are interested only in the full aggregation of route flows to total link flows. In these applications the issue of non-unique route flows is not relevant. In fact, it is quite rare to find practical applications where fully disaggregate route flows are needed. But there are quite a few applications where various different intermediate levels of aggregation are needed:

1. Class-specific flows in multi-class models
2. “Select Link” Analysis: determine the distribution of link flows by their origins and destinations
3. Estimation of OD flows from link flows
4. Derivation of OD flows for a subarea of a region, e.g. for micro-simulation
5. Average travel time and average distance per OD in a generalized cost assignment
6. License plates surveys:
   a. Validate model results against survey data;
   b. Design a survey to capture travelers at least twice, or as much travel as possible.

A different route flow solution may lead to different answers in each of these partially aggregated analyses. The overarching goal of this research is to examine the nature and magnitude of the differences in a practical setting, where existing tools are applied to networks that are used in practice. Specifically, this paper reports examples of select link analyses for a network that represents the Chicago region in 1990. Solutions produced by five commercially available assignment tools are presented: implementations of the FW algorithm in CUBE, EMME, and TransCAD; the Origin User Equilibrium (OUE) assignment in TransCAD; and the route-based assignment in VISUM. In addition we also present solutions produced by a research assignment tool called TAPAS (Bar-Gera, 2009). The motivation for including this research tool is discussed in section 2.

If multiple route flow solutions exist, the natural question is how to choose among them. It is common to attempt to answer this question by focusing on the criterion of realism. We believe that
models should be evaluated using a broader perspective of usefulness. A model is useful if it can help in practical decision processes. Obviously, empirical evidence demonstrating the realism of a certain model makes it more useful, assuming all else being equal. Other criteria for a model to be useful could be: data availability, stability, repeatability, consistency, computational efficiency, insight and plausibility, etc.

At present it seems that available empirical data are not sufficient to identify which UE route flow solution is the most realistic one. Yet this does not necessarily mean that all UE route flows are equally useful. Recent research shows that the issue of non-unique UE route flows can be resolved (almost completely) by a simple assumption, referred to as the condition of proportionality, to be presented in Section 2. This plausible condition ensures stability and consistency, thus leading to solutions that may be more useful than arbitrarily chosen solutions.

The second goal of this research is therefore to examine whether the condition of proportionality is satisfied in solutions produced by different assignment tools. A method to evaluate the condition of proportionality for each solution by itself is presented and applied.

2 PROPORTIONALITY

The condition of proportionality states that the same proportions apply to all travelers, regardless of their origins and destinations, who face a choice between a pair of alternative segments. The application of this condition to the example in Figure 1 is fairly straightforward. In this case travelers face a choice between segments [2, 3, 5] and [2, 4, 5]. Overall there are 40 vehicles on segment [2, 3, 5] and 120 vehicles on segment [2, 4, 5], so the proportion of segment [2, 3, 5] is 40 / (40+120) = 1/4. The condition of proportionality states that this proportion applies to travelers from origin A, therefore there should be 100 * 1/4 = 25 vehicles on route [A, 1, 2, 3, 5, C], as well as to travelers from origin B, therefore there should be 60 * 1/4 = 15 vehicles on route [B, 1, 2, 3, 5, C].

Figure 2 shows a slightly more complicated example of a network with three consecutive pairs of alternative segments. Considering route [1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14] that traverses the top segment in each pair, under the condition of proportionality its flow is 200 * (150/200) * (40/200) * (80/200) = 12. The application of the condition of proportionality in general networks is a bit more complicated, and discussed in detail in (Bar-Gera, 2006; 2009).

The main reasons to adopt proportionality are: (1) a reasonable condition that is easy to understand; (2) consistent treatment which may be important when equity issues are present; (3) provision of stable solutions with respect to model inputs (Lu and Nie, 2009); and (4) satisfaction of the condition of proportionality can be tested directly for any solution. These properties contribute to solution usefulness, especially since the only other existing alternative, at present, is to make a completely arbitrary choice.

An important implication of proportionality is that any route that can be used under the UE condition should be used. For example, in Figure 2 there are 8 routes; under proportionality all of them are used. So a precondition to satisfying proportionality is to make sure that “no route is left behind,” unless of course it is not a minimum cost route. We refer to this property of the set of routes as “consistency.” The consistency precondition is particularly important to the design of algorithms that aim to satisfy proportionality. It is also helpful for the interpretation of the results shown in Section 4.2.

TAPAS (Bar-Gera, 2009) is a UE assignment algorithm designed to identify route flow solutions that satisfy the condition of proportionality, and the precondition of route set consistency. Evaluation of solutions produced by TAPAS is therefore an integral component of this research.
3 TRAFFIC ASSIGNMENT PRECISION

UE convergence precision and the issue of route flows are in some ways fairly strongly intertwined, while in other ways they are completely orthogonal. In particular, as far as we know, not much has been done in practice regarding route flows. On the other hand, there has been a remarkable change, almost a revolution, regarding precision over the last five years or so. From our point of view, when we asked practitioners five years ago about the assignment algorithm, number of iterations and stopping condition they use, a typical answer was 10 iterations of the Frank-Wolfe (FW) algorithm. If we suggested that more iterations might be helpful, the typical response was that this would be a complete waste of computer time.

In the last year, we had the opportunity to interact with several practitioners, and discuss the issue of precision with them. The opinions have changed quite dramatically. Several hundred FW iterations are now quite typical, and solutions produced by 10 FW iterations are considered unsuitable for any meaningful analysis. Indeed, during the same five year period, major progress has been made in the assignment tools available commercially (Dial, 2006; Gentile, 2009; Slavin et al., 2009; Florian et al., 2009).

The level of UE convergence of a solution can be evaluated by an overall measure such as the relative gap. We note that the term "relative gap" is defined in several different ways, all leading to values within the same order of magnitude, which is sufficient for the purposes of this paper. More important than the overall measure of convergence are the deviations in predicted flows for individual links. These are depicted in Figure 3 for four solutions produced by FW in 10, 100, 1000, and 10000 iterations respectively. For each solution deviations in link flows are computed relative to an extremely precise reference solution produced by TAPAS, which is converged to a relative gap of 1E-12. Deviations for each solution are sorted, and plotted against the cumulative proportion from the total number of links.

The resulting graphics show for every deviation magnitude the proportion of the links that exhibit deviation of higher magnitude. For example the emphasized dot on the curve for 10 FW iterations indicates that a deviation of 10 vph or more, which is not trivial, occurs for 40% of the links. It seems likely that in many applications this precision would not be enough, and more iterations should be performed.

Theoretically the FW algorithm is proven to converge to the exact equilibrium, so as the number of iterations increases link flow deviations should decrease to zero. This is observed very nicely in Figure 3. Note that deviations are presented on a log scale, so each shift to the left from one solution to the next represents approximately a ten-fold reduction in the deviations.

The level of precision needed depends on the application. One of the main uses of travel forecasts is to compare scenarios. The comparison makes sense only if the precision of each solution is substantially better than the difference between the scenarios. Several case studies (Boyce et al., 2004; Slavin et al., 2009) suggest that a relative gap of 1E-4 may be a reasonable choice, although more case studies are needed to verify this rule of thumb.

For the purpose of this research we wanted to make sure that all the solutions are sufficiently converged in terms of link flows, so that residual deviations will not influence the evaluation of route flows and related analyses. Therefore, all evaluated solutions are solved to relative gap of 1E-4. A comparison of link flows with the reference solution (RG=1E-12) is presented in Figure 4, demonstrating that indeed link flows are nearly the same, and providing the necessary verification that the conversion of input data to all software has been performed properly.

It is important to point out that while all the solutions we are evaluating here have a similar level of precision, this does not mean that all the tools have the same performance, since
computation times needed to obtain the solutions were quite different. Performing convergence comparison between commercial software in a proper manner is a very delicate task that goes beyond the scope of this research. As an illustration for the potential differences in performance we show in Figure 5 a comparison of convergence vs. CPU time for three research tools: research implementation of the FW algorithm, OBA (Bar-Gera, 2002) and TAPAS (Bar-Gera, 2009). We can see that modest levels of precision can be obtained fairly quickly by all three methods. When higher precision is needed, the computational time for FW increases dramatically, while other methods can achieve high precision fairly quickly. We refer to such methods as “quick-precision” methods. Among the tools evaluated in this research, TransCAD- OUE and VISUM are considered as quick-precision methods.

4 RESULTS
4.1 Select Link Analyses
One of the most typical analyses of partially aggregated route flows is select link analysis. In this analysis we want to know the breakdown of a flow on a single link by OD pair. Figures 6-9 present comparisons between the six evaluated solutions and the reference solution. Each figure shows the OD flows through a different link in the Chicago network. OD flows on both axes are in log scale. Points along the axes represent values below 1E-4, including zeros. The left columns present the results for the three FW-type methods and the right columns present the results for the quick-precision methods.

In many cases practitioners examine the set of OD pairs that utilize a particular link. To see that the sets are different from solution to solution, the number of OD's are shown in the figures. For example in Figure 6 we see that the numbers of OD's that utilize North Ave. Bridge WB are 5165, 6614, 4202, 3061, 1975, 1507 for the evaluated solutions and 3057 for the reference solution. These numbers are quite different, showing that the sets of OD's are clearly not identical.

In other cases the focus is on flows, depicted by the X-Y comparisons in the plots. The three FW-type tools have a fairly similar performance, where the larger flows are quite similar to those in the reference solution (and therefore to each other), although there are quite a few OD pairs with small flows that should be zero according to the reference solution. These are mostly residual flows on suboptimal routes that should have been eliminated. The well known inability of the FW algorithm to remove these residual flows is a major component of the problematic convergence behavior of the algorithm.

Results for the quick precision tools exhibit non-trivial deviations over the entire range of flows. In particular there are many OD pairs that use the link according to the reference solution but do not use the link at all according to the evaluated solutions. These differences are mostly due to the issue of consistency mentioned in section 2, as there is a minimum cost route for the OD that traverses the link and could be used, which is ignored completely in the evaluated solutions for no particular reason.

This small sample of 4 links out of nearly 40,000 was chosen fairly arbitrarily, and is not necessarily “statistically representative.” Even so, it is enough to conclude that differences between solutions at the select link analysis level do occur.

4.2 Proportionality Evaluation
The condition of proportionality as stated in section 2 is based on identification of pairs of alternative segments. In the Chicago model there are about 5000 basic pairs of alternative segments, which can be used to construct all other pairs of alternative segments. We evaluated the condition of proportionality on two of them, results for the first pair of segments near Lake Shore.
Drive are shown in Figure 10, and results for the second pair of segments near North Ave. Bridge are shown in Figure 11. In each evaluated solution we found the breakdown of the flow on the two segments by OD. Each point represents a single OD. The horizontal and vertical axes represent the flows on segments 1 and 2 respectively. If the same proportions apply to all OD pairs, all the points should fall on a straight line (with a slope of 45 degrees). Both axes are in log scale. Points along the axes represent values below $1E^{-4}$, including zeros.

A fairly straight line is observed for all FW-type solutions, especially for higher flows. The plot for the evaluated TAPAS solution ($RG = 1E^{-4}$) in Figure 10 (near Lake Shore Drive) is also fairly close to a straight line. In the results from TransCAD-OUE for the same pair (Figure 10) we can see three main lines; each line corresponds to a different origin. This means that within each origin proportionality is maintained, but between origins proportions are not the same. The VISUM solution in this case is quite extreme, where only one OD pair uses both segments.

In the case of the second pair of segments near North Ave. Bridge, proportionality is reasonably maintained by the FW-type methods, but not for the three quick-precision methods. In all three methods there are OD pairs that use only one segment of the two, which is probably chosen completely arbitrarily. This is a clear issue of consistency in the set of used routes. For TransCAD-OUE and VISUM, even for OD pairs that use both segments, the proportions are different from one OD to the next. In the case of the evaluated TAPAS solution ($1E^{-4}$), OD pairs that use both segments fall along a single line, meaning that the proportions for those OD pairs are the same. In other words, the evaluated TAPAS solution has limited consistency, but it does satisfy proportionality within this limitation.

Figure 12 shows that consistency in TAPAS solutions improves considerably with convergence. Nearly perfect consistency is shown here at relative gap around $1E^{-9}$, and a noticeable progress is shown already at relative gap around $1E^{-7}$. Notice that reaching these higher levels of precision does not require too much computation time. Preliminary experiments with commercial quick-precision tools did not demonstrate improvement in consistency or proportionality at higher levels of precision, but additional exploration is needed to verify these observations.

The two pairs of alternative segments do not necessarily represent all 5000 other pairs in this model. They do offer an idea for what might be expected in other cases.

**CONCLUSIONS**

This research compared the results of select link analyses in several commercially available traffic assignment tools, and evaluated the extent to which they satisfy the condition of proportionality. The results show that select link analysis results are different between solution in a meaningful and disturbing manner. Solutions produced by the FW algorithm tend to satisfy the condition of proportionality to a reasonable extent, although not perfectly. Alternative methods offer important benefits in terms of quick-precision, which are highly valuable for scenario comparisons, but at least the tools evaluated here exhibit noticeable and arbitrary deviations from the condition of proportionality. The results of such methods in select link and other analyses that are based on route flows should be used with extreme caution.

Evaluation of the TAPAS research tool shows that it provides quick precision, reasonable consistency particularly at higher level of convergence, and satisfaction of the condition of proportionality.

Future research could follow three main directions: additional evaluation of the differences between route flow solutions in other types of analyses or in other case studies; alternative
approaches to satisfy route set consistency and proportionality in UE assignment; and empirical studies to evaluate the validity of the condition of proportionality in reality.

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REFERENCES
FIGURE 1 Multiple UE Route Flow Solution Example

FIGURE 2 Proportionality Example
FIGURE 3 Distribution of Deviations in Link Flows by Solution Precision
Chicago Regional Network
FIGURE 4 Comparison of Total Link Flows in Evaluated Solutions (RG=1E-4) to Reference TAPAS Solution (RG=1E-12)
FIGURE 5 Convergence Performance Comparison of Research Tools
Chicago Regional Network
Each point represents vehicle flow per hour for one OD pair.
X - reference solution (TAPAS-3,057 ODs) ; Y - evaluated solution (RG = 1e-4)
FIGURE 7 OD Flows through North Ave. Bridge EB

Each point represents vehicle flow per hour for one OD pair.
X - reference solution (TAPAS-3,376 ODs) ; Y - evaluated solution (RG = 1e-4)
FIGURE 8 OD Flows through Harlem Ave. SB

Each point represents vehicle flow per hour for one OD pair. X - reference solution (TAPAS-4,752 ODs) ; Y - evaluated solution (RG = 1e-4)
FIGURE 9 OD Flows through Harlem Ave. NB

Each point represents vehicle flow per hour for one OD pair.
X - reference solution (TAPAS- 5,034 ODs) ; Y - evaluated solution (RG = 1e-4)
FIGURE 10 Paired Alternative Segments near Lake Shore Drive

Each point represents vehicle flow per hour for one OD pair.
X - Segment 1; Y - Segment 2; all solutions converged to RG = 1e-4.
FIGURE 11 Paired Alternative Segments near North Ave. Bridge

Each point represents vehicle flow per hour for one OD pair.
X - Segment 1; Y - Segment 2; all solutions converged to RG = 1e-4.
FIGURE 12 Paired Alternative Segments near North Ave. Bridge
Effect of convergence on consistency in TAPAS

Each point represents vehicle flow per hour for one OD pair.
X - Segment 1; Y - Segment 2; all solutions converged to RG = 1e-4.